Compression for Multiple Reconstructions

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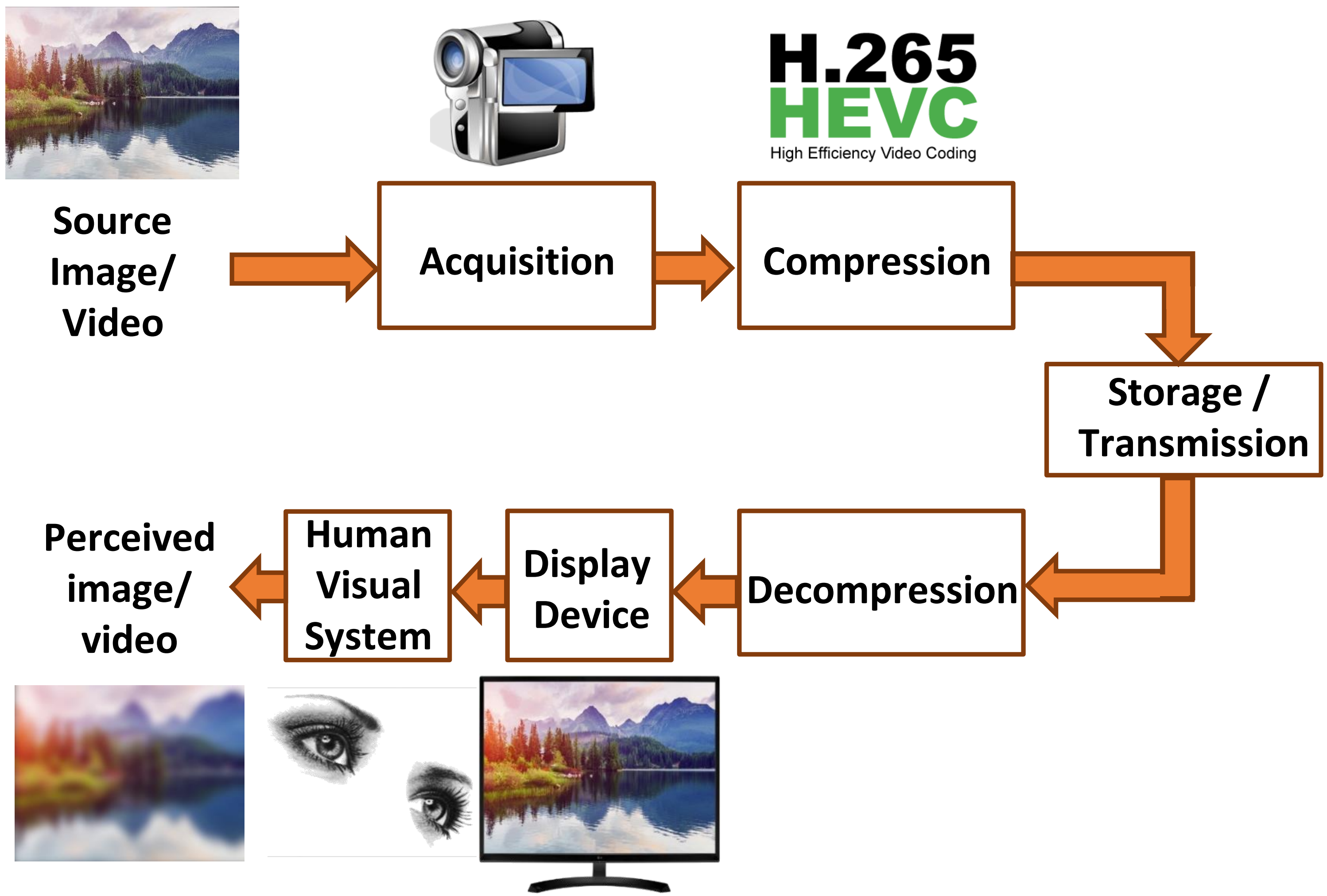
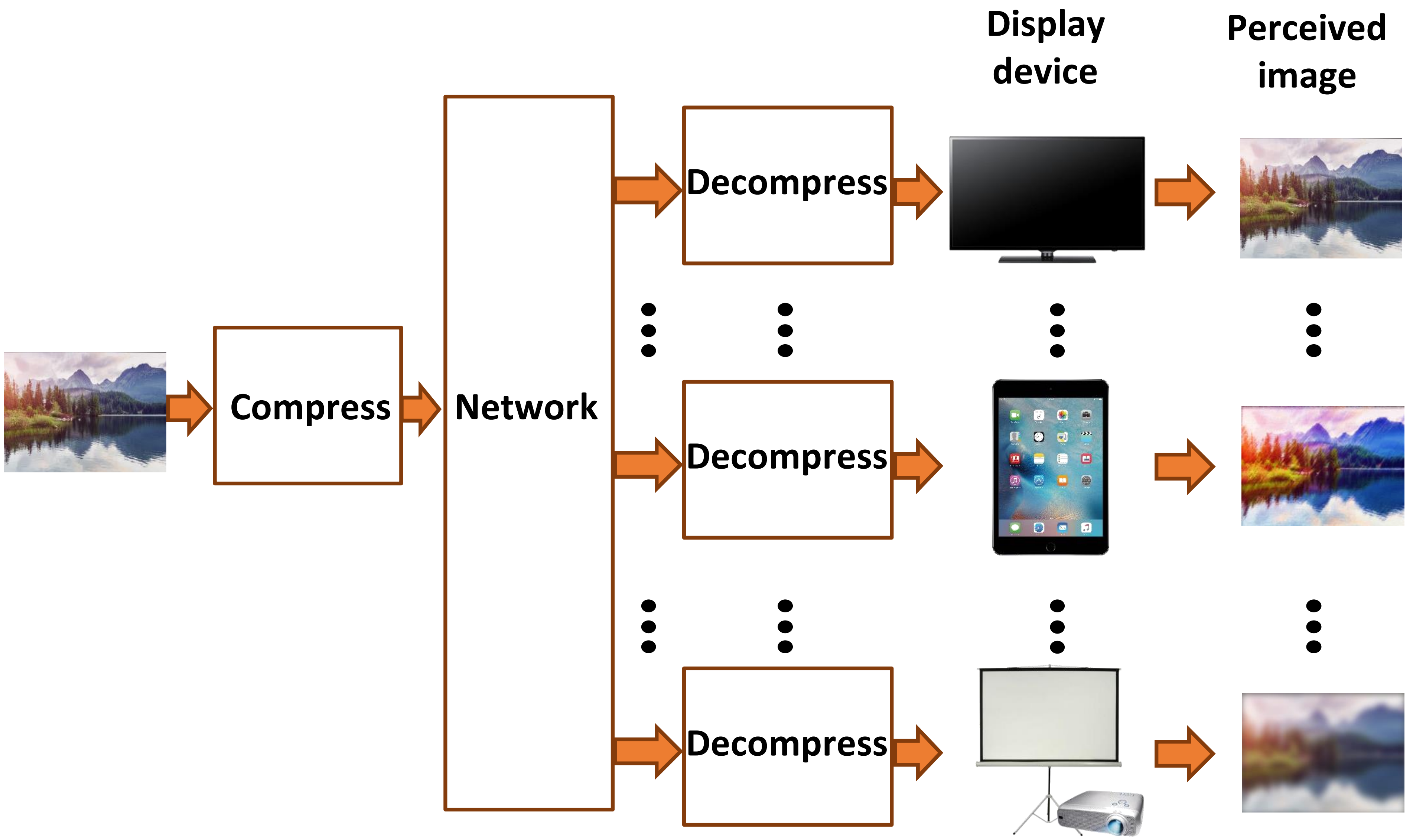
Motivation: Imaging Systems

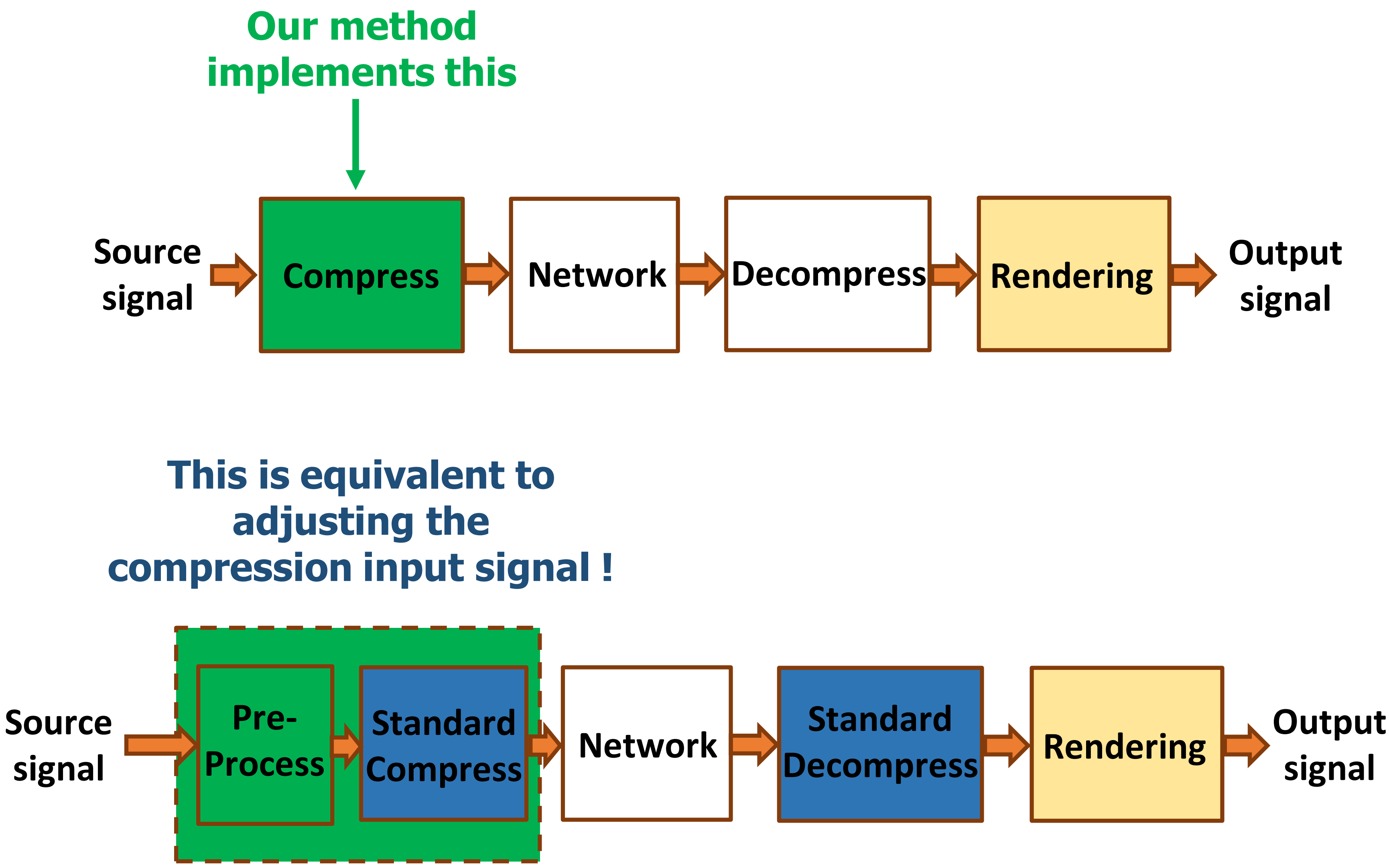


Standard compression techniques ignore the system structure!

Motivation:
Multimedia Distribution Networks



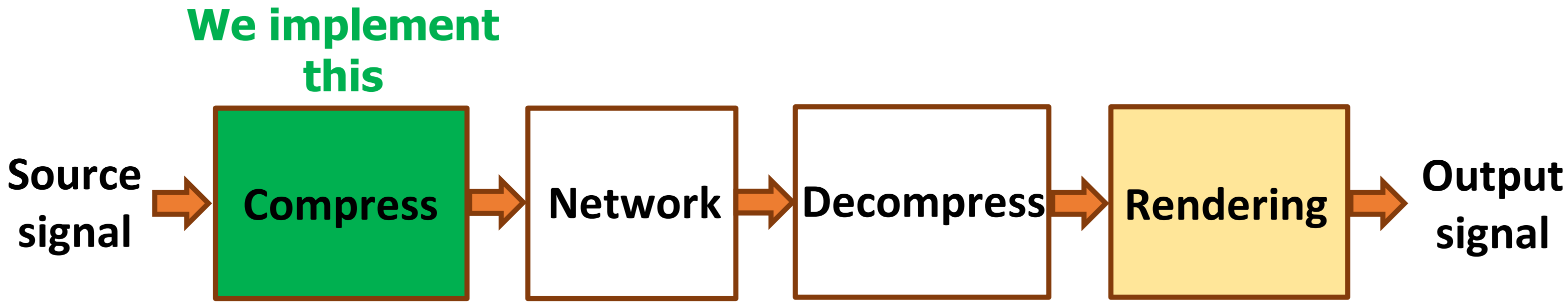
The Proposed Compression Method



The Proposed Compression Method

Desired Properties:

- **Provide compressed signals that compensate the degradations without any post-filtering.**
- **Optimize the end-to-end distortion with respect to the bit cost.**
- **Generically leverage existing compression techniques.**



The Proposed Compression Method: The Challenge

Requirement #1:

Using **standard** compression techniques.

Problem

Ignore the network structure → **suboptimal !**

Requirement #2:

Having **network-optimized** compression.

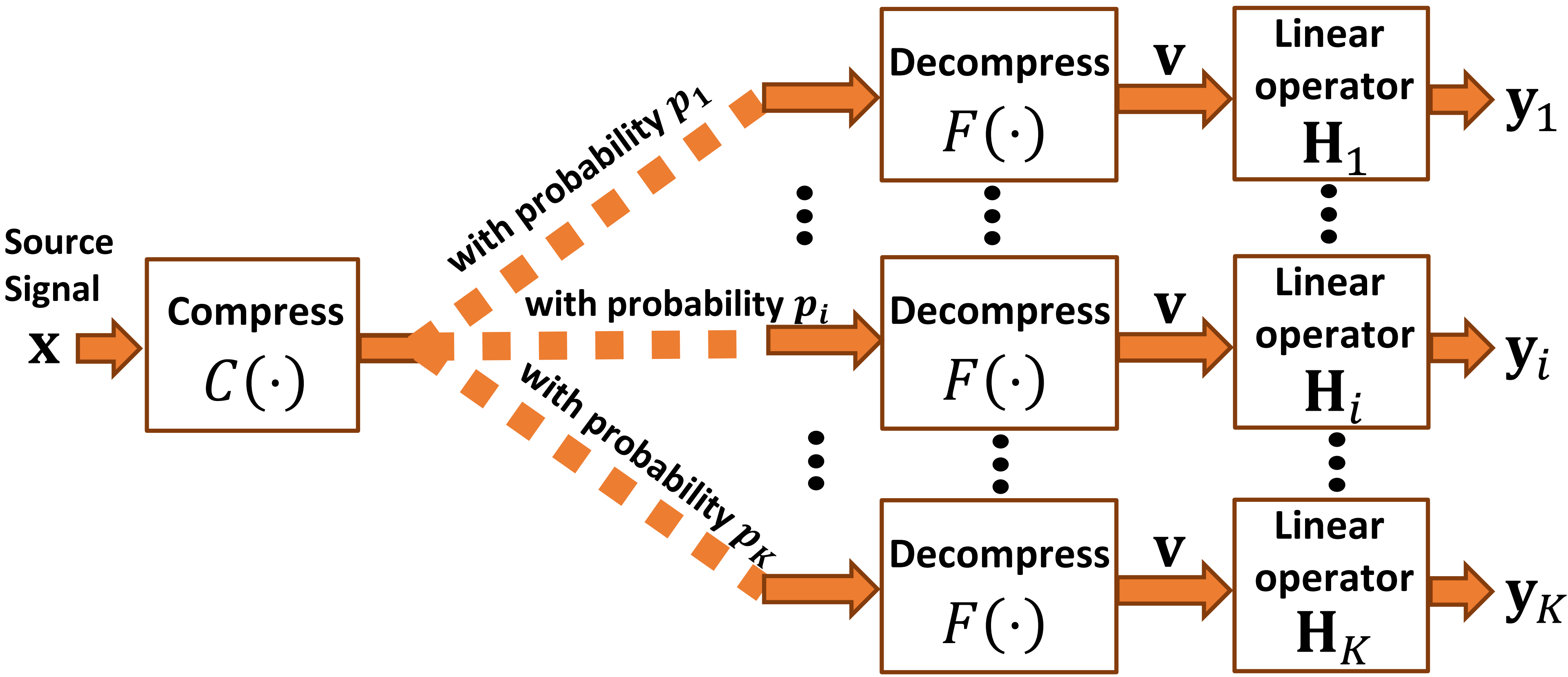
Problem

Needs a **network-specific** design.

The proposed framework:

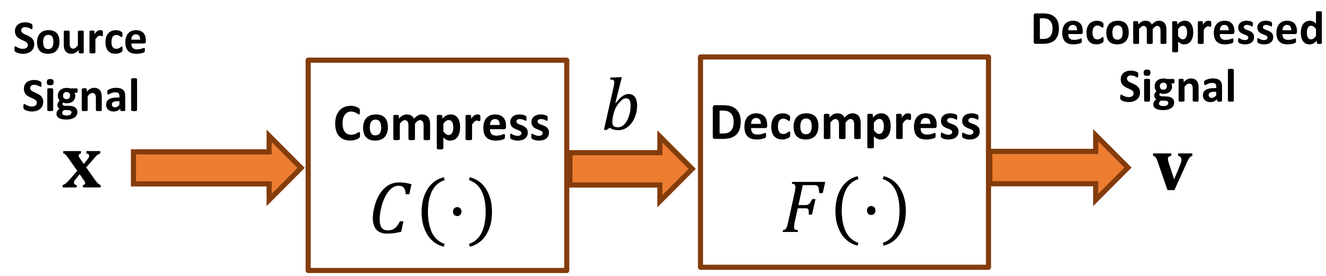
Extends **standard compression** techniques to **network-optimized** compression procedures.

Model and Problem Formulation:
Network Structure



The display devices are modeled using the matrices $\mathbf{H}_1, \ldots, \mathbf{H}_K$.

Model and Problem Formulation:
Compression Architecture



$$C\colon \mathbb{R}^N \rightarrow \mathcal{B} \qquad F\colon \mathcal{B} \rightarrow \mathcal{S}$$

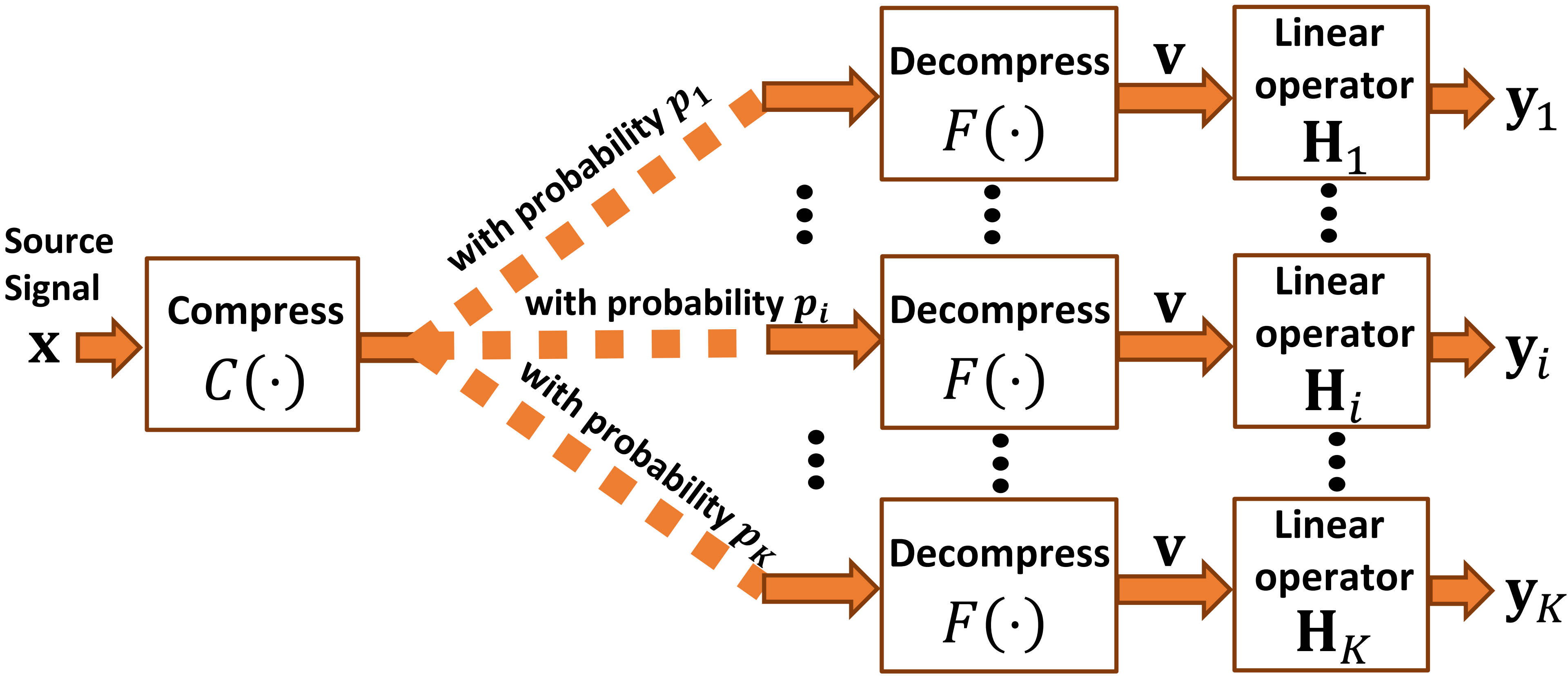
$$b = C(\mathbf{w}) \qquad \mathbf{v} = F(b)$$

$\mathcal{B}$ is a **discrete set** of **binary compressed** representations

$\mathcal{S} \subset \mathbb{R}^N$ is a **discrete set** of **decompressed signals**

For $\mathbf{v} \in \mathcal{S}$: $R(\mathbf{v})$ evaluates the compressed **binary length** of $b = F^{-1}(\mathbf{v})$

Model and Problem Formulation:
Distortion Metric



Network distortion:

$$D(\mathbf{x},\mathbf{v}) = \frac{1}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{v}\|_2^2$$

Model and Problem Formulation:
Network-Aware Compression Optimization

Our compression goal:

Minimize the **bit-cost** under a **network distortion constraint**

$$\hat{\mathbf{v}} = \underset{\mathbf{v} \in \mathcal{S}}{\operatorname{argmin}} \, R(\mathbf{v})$$

$$\text{s.t.} \; \frac{1}{N} \sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i \mathbf{v}\|_2^2 \leq D$$

An operational rate-distortion optimization for the input signal $\mathbf{x}$ with respect to the compression architecture $\{\mathcal{S}, R\}$.

The corresponding compression mapping: $C(\mathbf{x}) \triangleq F^{-1}(\hat{\mathbf{v}})$

Model and Problem Formulation: System-Aware Compression Optimization

The unconstrained **Lagrangian** optimization:

$$\hat{\mathbf{v}} = \underset{\mathbf{v}\in\mathcal{S}}{\operatorname{argmin}} \; R(\mathbf{v}) + \lambda \frac{1}{N} \sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i \mathbf{v}\|_2^2$$

where λ is a Lagrange multiplier corresponding to some distortion level D_λ

Computationally hard due to $\mathbf{H}_1, \dots, \mathbf{H}_K$!

The Variable Splitting Trick

The hard optimization:

$$\hat{\mathbf{v}} = \underset{\mathbf{v}\in\mathcal{S}}{\operatorname{argmin}}\ R(\mathbf{v}) + \frac{\lambda}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{v}\|_2^2$$

Two complicated parts considering $\mathbf{v}$

Variable splitting:

$$(\hat{\mathbf{v}}, \hat{\mathbf{z}}) = \underset{\mathbf{v}\in\mathcal{S}\ ,\ \mathbf{z}\in\mathbb{R}^N}{\operatorname{argmin}}\ R(\mathbf{v}) + \frac{\lambda}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{z}\|_2^2$$

subject to $\mathbf{v} = \mathbf{z}$

where $\mathbf{z}$ is an additional vector due to the split.

Design Overview

The ADMM Optimization Form

Augmented Lagrangian (scaled version) and the **Method of Multipliers**:

$$\left(\hat{\mathbf{v}}^{(t)}, \hat{\mathbf{z}}^{(t)}\right) = \underset{\mathbf{v}\in\mathcal{S}\ ,\ \mathbf{z}\in\mathbb{R}^M}{\operatorname{argmin}}\ R(\mathbf{v}) + \frac{\lambda}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{z}\|_2^2 + \frac{\beta}{2}\left\|\mathbf{v} - \mathbf{z} + \mathbf{u}^{(t)}\right\|_2^2$$

$$\mathbf{u}^{(t+1)} = \mathbf{u}^{(t)} + \left(\hat{\mathbf{v}}^{(t)} - \hat{\mathbf{z}}^{(t)}\right)$$

where $\mathbf{u}^{(t)} \in \mathbb{R}^M$ is the scaled dual-variable and
β is a parameter introduced in the augmented Lagrangian.

Alternating minimization provides the **ADMM form**:

$$\hat{\mathbf{v}}^{(t)} = \underset{\mathbf{v}\in\mathcal{S}}{\operatorname{argmin}}\ R(\mathbf{v}) + \frac{\beta}{2}\left\|\mathbf{v} - \tilde{\mathbf{z}}^{(t)}\right\|_2^2$$

$$\hat{\mathbf{z}}^{(t)} = \underset{\mathbf{z}\in\mathbb{R}^N}{\operatorname{argmin}}\ \frac{\lambda}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{z}\|_2^2 + \frac{\beta}{2}\left\|\mathbf{z} - \tilde{\mathbf{v}}^{(t)}\right\|_2^2$$

$$\mathbf{u}^{(t+1)} = \mathbf{u}^{(t)} + \left(\hat{\mathbf{v}}^{(t)} - \hat{\mathbf{z}}^{(t)}\right)$$

where

$$\tilde{\mathbf{z}}^{(t)} = \hat{\mathbf{z}}^{(t-1)} - \mathbf{u}^{(t)}$$

$$\tilde{\mathbf{v}}^{(t)} = \hat{\mathbf{v}}^{(t)} + \mathbf{u}^{(t)}$$

Leveraging a Standard Compression

$$\hat{\mathbf{v}}^{(t)} = \underset{\mathbf{v}\in\mathcal{S}}{\operatorname{argmin}}\, R(\mathbf{v}) + \frac{\beta}{2}\left\|\mathbf{v} - \tilde{\mathbf{z}}^{(t)}\right\|_2^2$$

$$\hat{\mathbf{z}}^{(t)} = \underset{\mathbf{z}\in\mathbb{R}^N}{\operatorname{argmin}}\, \frac{\lambda}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{z}\|_2^2 + \frac{\beta}{2}\left\|\mathbf{z} - \tilde{\mathbf{v}}^{(t)}\right\|_2^2$$

$$\mathbf{u}^{(t+1)} = \mathbf{u}^{(t)} + \left(\hat{\mathbf{v}}^{(t)} - \hat{\mathbf{z}}^{(t)}\right).$$

Compression optimization considering **regular squared error**

We suggest to

replace this step with an application of a **standard compression method**

Design Overview

Leveraging a Standard Compression

$$b^{(t)} = StandardCompress\left(\tilde{\mathbf{z}}^{(t)}, \rho\right)$$

$$\hat{\mathbf{v}}^{(t)} = StandardDecompress\left(b^{(t)}\right)$$

$$\hat{\mathbf{z}}^{(t)} = \underset{\mathbf{z}\in\mathbb{R}^N}{\text{argmin}} \frac{\lambda}{N}\sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{z}\|_2^2 + \frac{\beta}{2}\left\|\mathbf{z} - \tilde{\mathbf{v}}^{(t)}\right\|_2^2$$

$$\mathbf{u}^{(t+1)} = \mathbf{u}^{(t)} + \left(\hat{\mathbf{v}}^{(t)} - \hat{\mathbf{z}}^{(t)}\right).$$

Compression optimization considering **regular squared error**

We suggest to

replace this step with an application of a **standard compression method**

ℓ_2-Constrained Deconvolution

$$b^{(t)} = StandardCompress\left(\tilde{\mathbf{z}}^{(t)}, \theta\right)$$

$$\hat{\mathbf{v}}^{(t)} = StandardDecompress\left(b^{(t)}\right)$$

$$\hat{\mathbf{z}}^{(t)} = \underset{\mathbf{z}\in\mathbb{R}^N}{\operatorname{argmin}} \frac{\lambda}{N} \sum_{i=1}^{K} p_i \|\mathbf{x} - \mathbf{H}_i\mathbf{z}\|_2^2 + \frac{\beta}{2} \left\|\mathbf{z} - \tilde{\mathbf{v}}^{(t)}\right\|_2^2$$

$$\mathbf{u}^{(t+1)} = \mathbf{u}^{(t)} + \left(\hat{\mathbf{v}}^{(t)} - \hat{\mathbf{z}}^{(t)}\right).$$

Extension of ℓ_2-Constrained Deconvolution

Optimization of simple quadratic terms

This can be easily solved!

The Proposed Approach: Concept

The complicated optimization: $\hat{\mathbf{v}} = \underset{\mathbf{v} \in \mathcal{S}}{\operatorname{argmin}} \; R(\mathbf{v}) + \frac{\lambda}{N} \sum_{i=1}^{K} p_i \| \mathbf{x} - \mathbf{H}_i \mathbf{v} \|_2^2$

is addressed via **repeated** application of **3 simple procedures**:

associated with

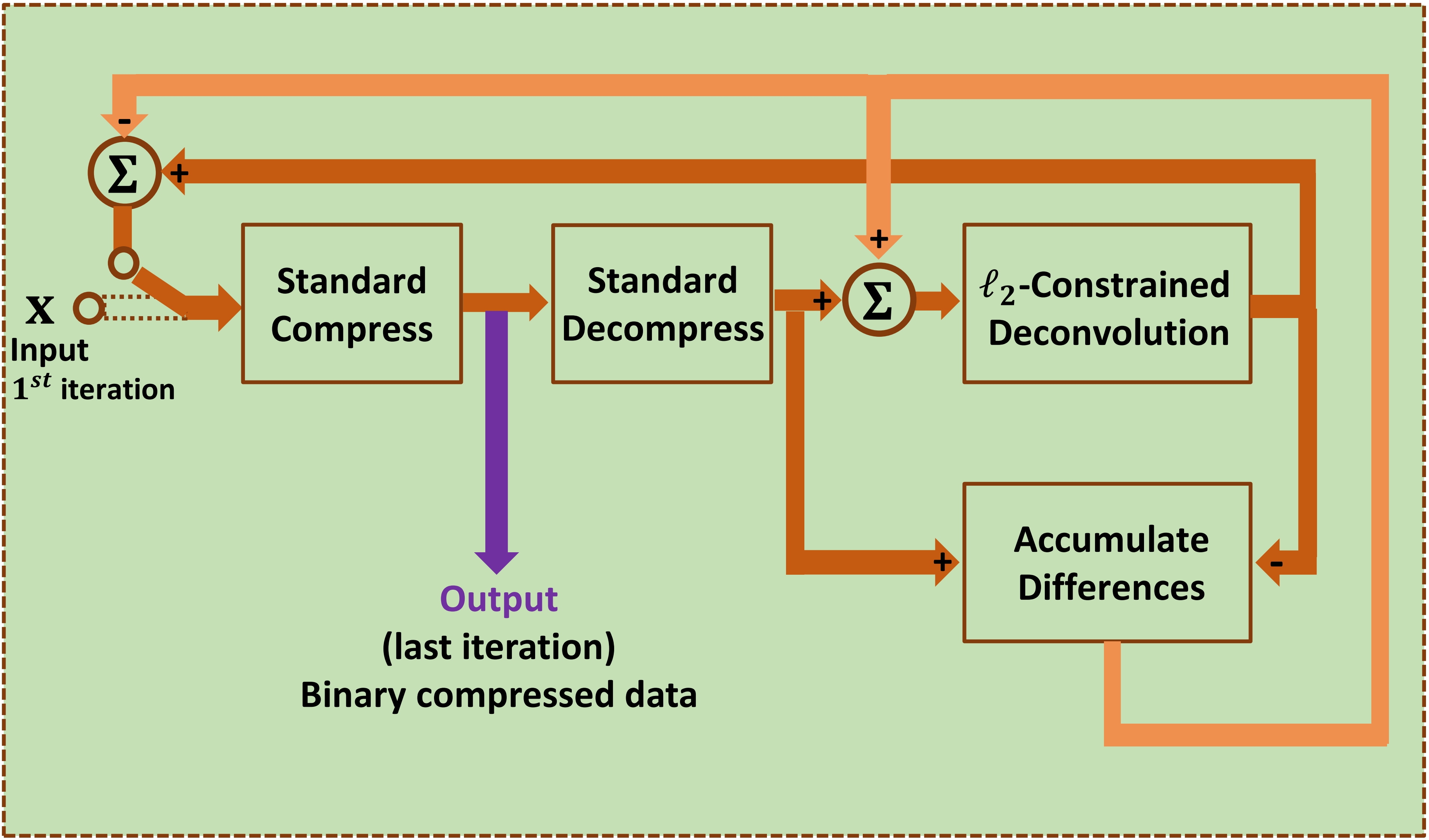
$$\hat{\mathbf{v}} = \underset{\mathbf{v} \in \mathcal{S}}{\operatorname{argmin}} \; R(\mathbf{v}) + \frac{\lambda}{N} \| \mathbf{x} - \mathbf{v} \|_2^2$$

conceptually employed in modern compression techniques

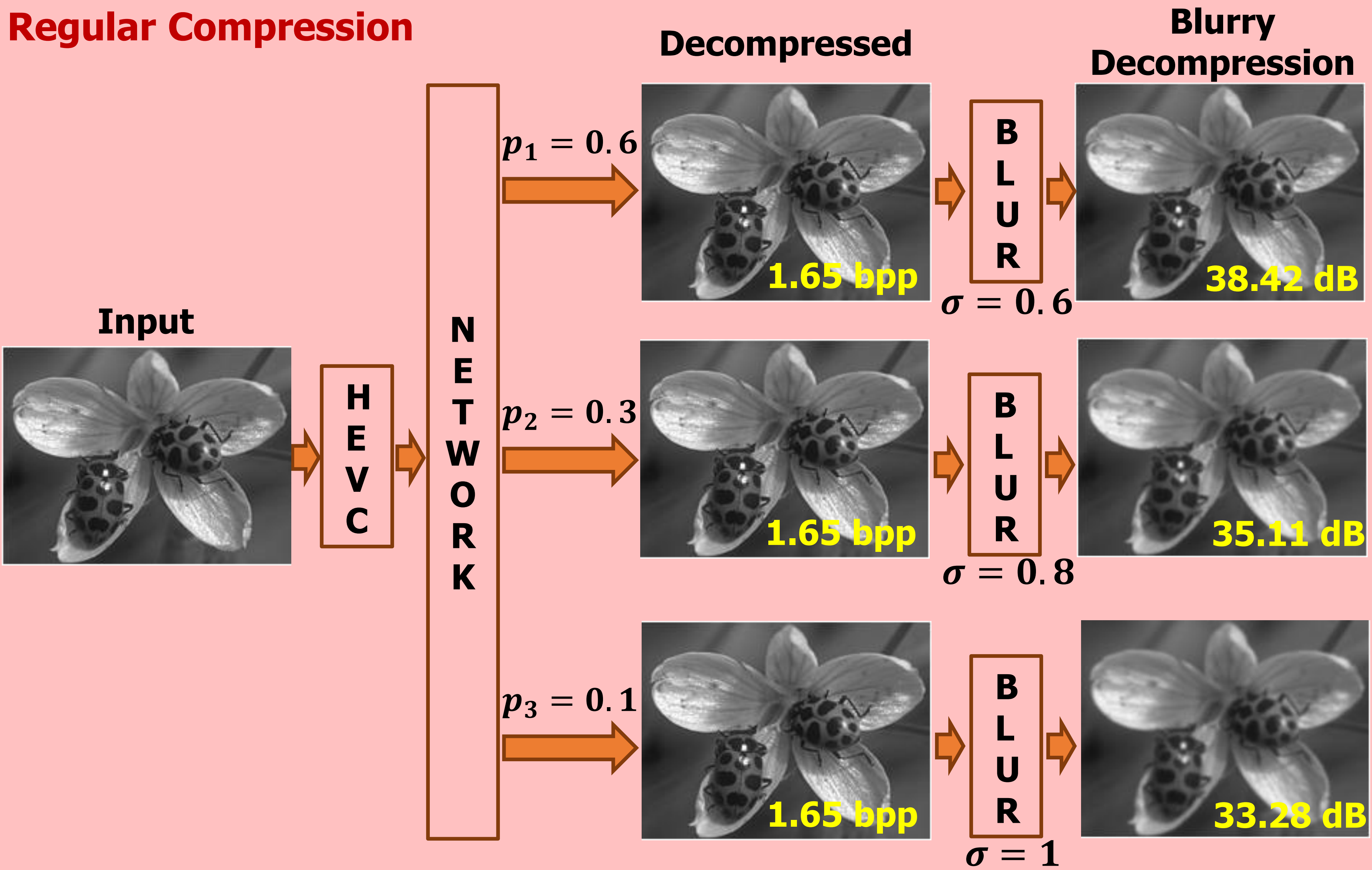
- **Standard Compression** — Examples: HEVC/H.265, JPEG2000, …
- **Standard Decompression**
- **Simple Deconvolution** — If $\{\mathbf{H}_i\}_{i=1}^{K}$ blur, this stage sharpens

Design Overview

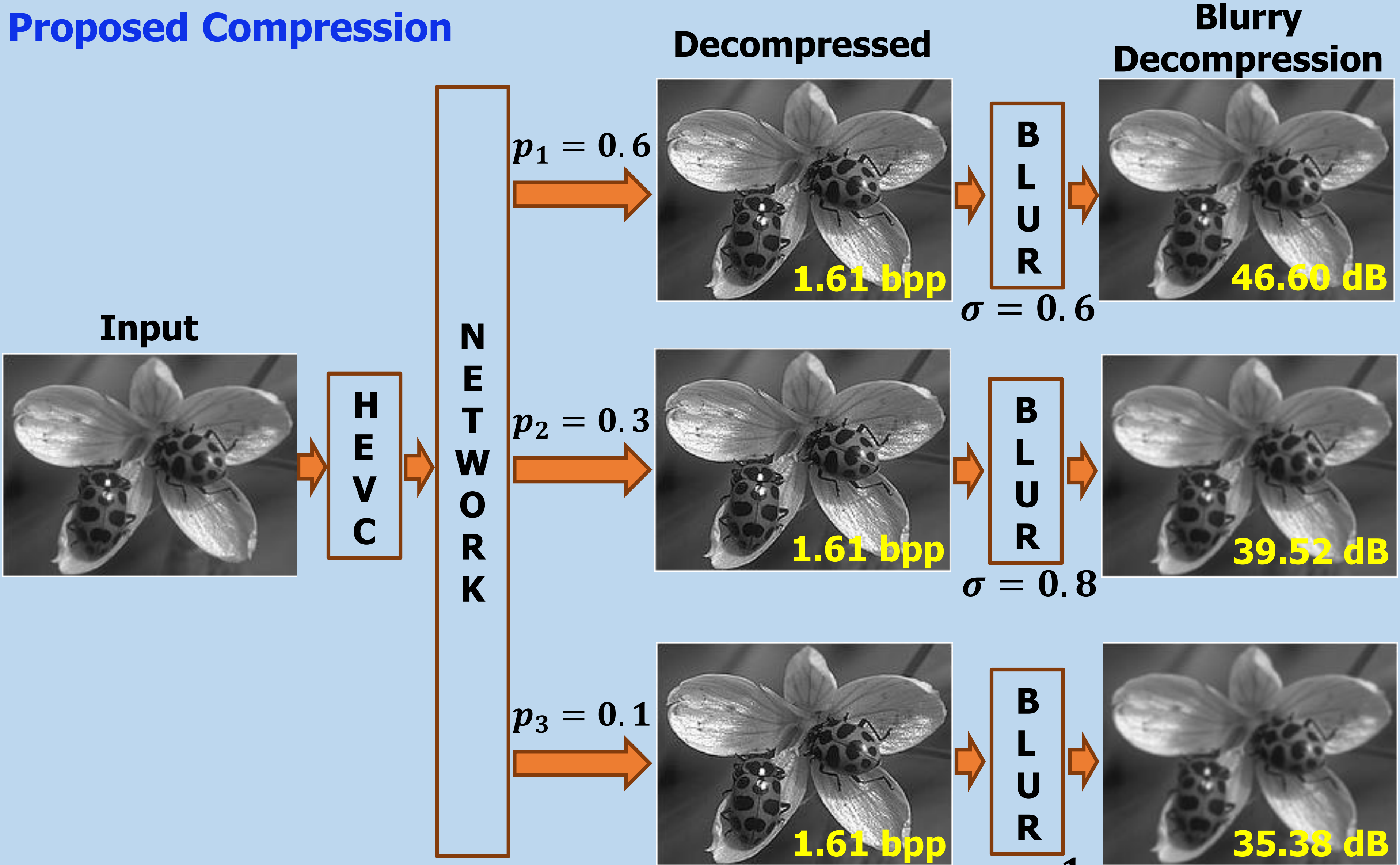
The Proposed Compression Method



Experiments: HEVC + Gaussian Blur (three types)



Experiments: HEVC + Gaussian Blur (three types)



Experiments: HEVC + Gaussian Blur (three types)

Regular

1.65 bpp

Blurry Decompression

$\sigma = 0.6$

$\sigma = 0.8$

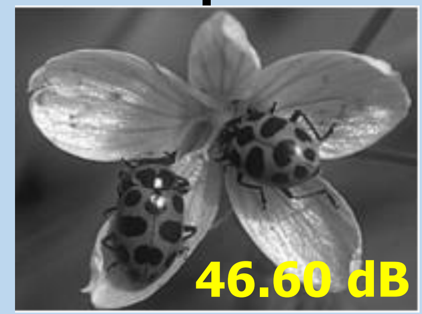
$\sigma = 1$

Proposed

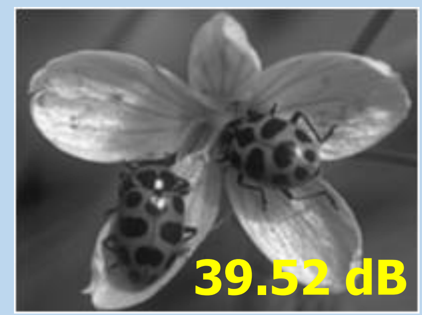
1.61 bpp

Blurry Decompression

$\sigma = 0.6$

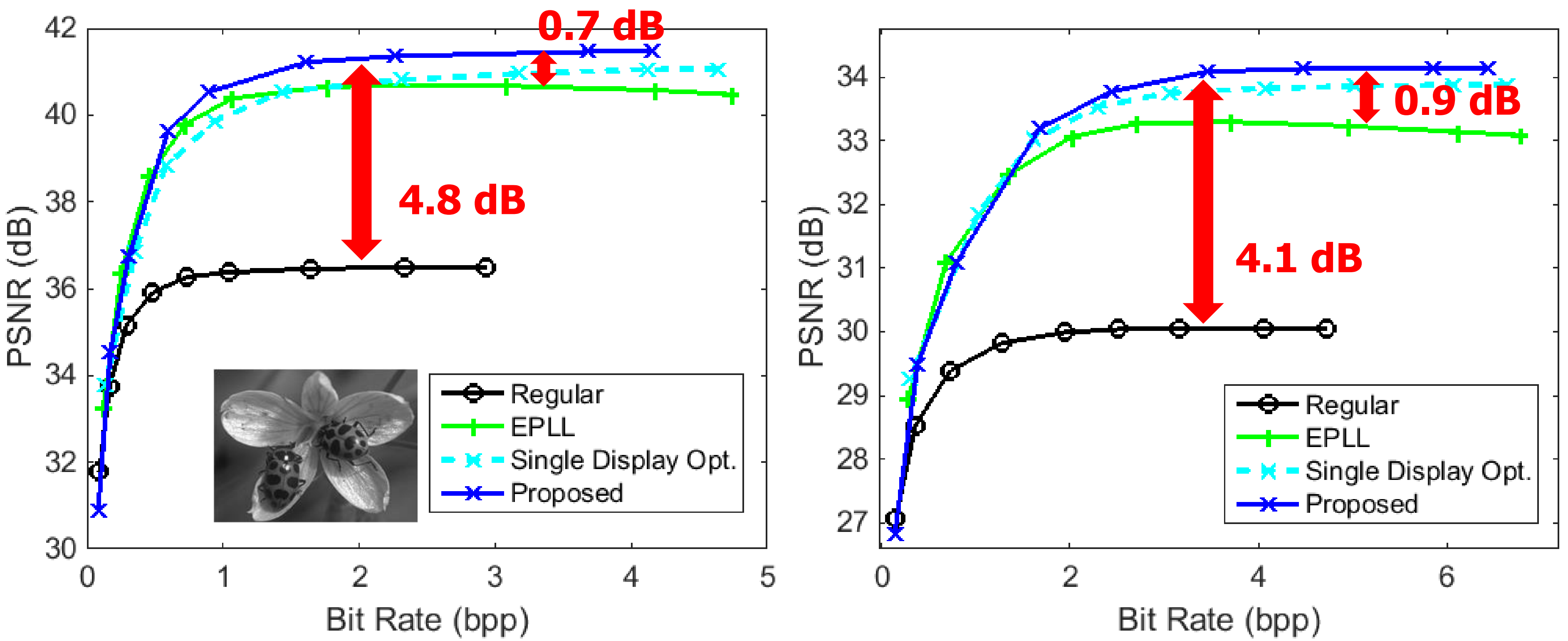


$\sigma = 0.8$



$\sigma = 1$

Experiments: HEVC + Gaussian Blur (three types)



- 12 images were evaluated.
- We obtain average **PSNR gains at high bit-rates**:
  - 0.5 – 1.4 dB over pre-compression EPLL deblurring [Zoran & Weiss, ICCV 2011].
  - 0.2 – 0.5 dB over single display optimization [Dar et al., IEEE TIP 2018].
  - **3.2 – 5.2 dB over the regular approach.**

Conclusions

We proposed a compression method for optimizing **network** structures involving **post-decompression degradations**.

- Applications to **HEVC** compression that compensates several post-decompression **blur** degradations.

A framework for **intricate** rate-distortion optimizations.

For related problems and analysis, see our papers:

- **"Optimized Pre-Compensating Compression"**, IEEE TIP, 2018.
- **"Restoration by Compression"**, IEEE TSP, 2018.
- **"System-Aware Compression"**, IEEE ISIT, 2018.
- **"Compression for Multiple Reconstructions"**, IEEE ICIP, 2018.